2D Electro–Optic Probing Combined with Field Theory Based Multimode Wave Amplitude Extraction: A New Approach to On-Wafer Measurement

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Abstract

A novel approach to on-wafer measurement is proposed, which combines the *direct electro-optic probing technique* with a *field theory based extraction technique for the modal voltages* of all relevant modes at arbitrary internal ports of a (III-V) MMIC. The approach can be extended to obtain the complex modal amplitudes of forward and backward propagating waves on interconnecting transmission lines. This makes it a unique method for measurement of mode conversion in N-port components and their multimode s-parameter characterization.

Introduction

On-wafer NWA–measurement using *coplanar probes* is the state of the art technique for characterization of active and passive MMIC components. This technique, however, has some fundamental limitations:

(a) It is basically limited to single mode measurements; even–odd mode conversion in CPW components is not accessible.

(b) Due to mechanical size of probes and quality of coaxial interconnects the technique breaks down for upcoming applications far above 100 GHz.

(c) Internal ports are not accessible. This makes it impossible to measure active device performance in its true circuit environment and under true bias conditions.

(d) Characterization of multi-port components is extremely difficult.

A new approach to in-circuit measurement is proposed here which promises to overcome these problems. It is a combination of the *direct electro-optic probing technique* with a field theory based *wave amplitude extraction technique*. Electro-optic probing has previously been demonstrated to be a powerful tool for characterization of high frequency signals in MMICs [1, 2]. It allows for less than picosecond time resolution and spatial resolution down to a few micrometers. Although the technique has found considerable interest in recent years its usefulness for the microwave engineer has not been adequately anticipated, because of the missing link to conventional circuit analysis. The circuit designer is interested in the complex modal voltages at internal ports, the complex amplitudes of forward and backward propagating waves on interconnecting transmission lines and the corresponding s-parameter characterization of circuit components. The *extraction technique* which is presented here provides the missing link.

Measurement setup

The measurement set-up shown in Fig. 1 was already described in more detail in [2]. The polarized laser beam of a picosecond Nd:YAG laser is focused from the substrate backside onto the front surface of the CPW structure. The local electric field in the illuminated region of the substrate modulates the polarization of the optical radiation due to the linear electro-optical effect (Pockels-effect). The reflected optical radiation is separated by a beamsplitter and passes a polarizer which converts the polarization change into a change of the optical intensity. The intensity modulation is

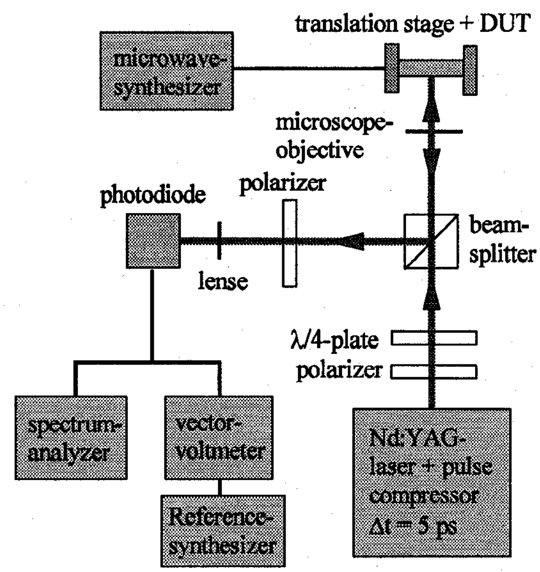

Fig. 1: Measurement setup.

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then detected by a photodiode and recorded in amplitude and phase by a spectrum-analyzer rsp. a vectorvoltmeter. Using ultrashort optical pulses the electro-optic measurement system is able to resolve electrical signals in the picosecond range which corresponds to a bandwidth exceeding 100 GHz.

Extraction Technique

A direct electro-optic probing system delivers a signal which, apart from a conversion factor and neglecting finite lateral resolution, is equal to the complex voltage

$$v(x,z) = \int_{y=-h}^{0} \hat{\mathbf{y}}\,\mathbf{E}(x,y,z)\,\mathrm{d}\,y \qquad (1)$$

(Fig. 2). Obviously this is not sufficient information for a complete characterization of the electromagnetic field because TE_y field contributions are not accessible. It is nevertheless possible to extract all quantities which are of relevance in conventional circuit analysis. The reason is, that the latter is concerned only with *projections* of the total electromagnetic field on sets of transmission line modes which are assumed to exist at internal ports of a circuit, e.g. the CPW even and odd mode. Thus circuit level descriptions are conceptually based on local field expansions of the form

$$\mathbf{E}(x,y,z) = \sum_{i=1}^{n} v_i^{\pm}\mathbf{e}_i(x,y)\,\mathrm{e}^{\mp\gamma_i z} + \mathbf{E}_\mathrm{r}(x,y,z) \qquad (2)$$

where the $\mathbf{e}_i(x,y)$ denote the transverse modal fields (normalized to unity modal voltage), $v_i^{\pm}$ the complex wave amplitudes and $\gamma_i \in \mathbb{C}$ the corresponding propagation constants. $\mathbf{E}_\mathrm{r}(x,y)$ consumes neglected higher order modes and any additional field contribution which is not contained in the discrete spectrum. The quantities of interest are the wave amplitudes and the total modal voltages

$$v_i(z) = v_i^{+}\,\mathrm{e}^{-\gamma_i z} + v_i^{-}\,\mathrm{e}^{+\gamma_i z} \qquad (3)$$

in a given referenceplane $z = \text{const}$.

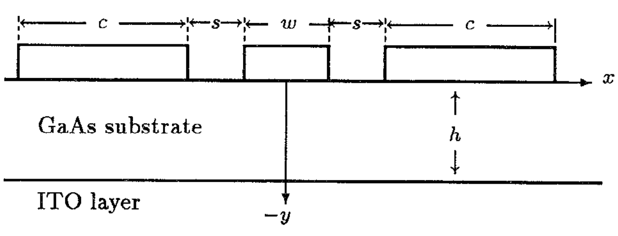

Fig. 2: Coordinate system, measurement environment and CPW geometry as used for the experimental results: $w = 44.8\,\mu\mathrm{m}, s = 28.8\mu\mathrm{m}, c = 200\mu\mathrm{m}, h = 500\,\mu\mathrm{m}$.

Within this picture an (ideal) coplanar probe attached to a NWA may be considered as a device which projects the total electromagnetic field onto the amplitudes $v_\mathrm{e}^{\pm}$ of the fundamental CPW even mode. The direct electro-optic probing technique can do better. On a section of homogeneous transmission line anywhere in a circuit we may substitute (2) for the electric field in (1). Introducing the *normalized transverse modal profiles*

$$u_i(x) = \int_{y=-h}^{0} \hat{\mathbf{y}}\,\mathbf{e}_i(x,y)\,\mathrm{d}\,y \qquad (4)$$

the electro-optic signal (1) of a measurement system with infinite lateral resolution may be written as

$$v(x,z) = \sum_{i=1}^{n} v_i(z)u_i(x) + v_\mathrm{r}(x,z) \qquad (5)$$

with $v_\mathrm{r}(x,z)$ corresponding to $\mathbf{E}_\mathrm{r}(x,y,z)$.

The focal plane beam diameter of the real measurement system is approximately $2\rho = 5\,\mu\mathrm{m}$ (Gaussian profile) and the beam forms a cone of less than 0.2° aperture, i.e. it is an almost perfect cylinder. Because $2\rho_0\gamma_i$ is always very small we may safely ignore the longitudinal field variation when correcting for finite spatial resolution. It is sufficient therefore to replace the transverse profiles $u_i(x)$ in (5) by

$$u_i(x) = \int_{\xi} g(x-\xi) \int_{y=-h}^{0} \hat{\mathbf{y}}\,\mathbf{e}_i(\xi,y)\,\mathrm{d}\,y\,\mathrm{d}\,\xi \qquad (6)$$

where $g(x)$ is the beam profile.

Extraction of modal voltages

Equation (5) is the starting point for the first step of the new extraction technique which deals with the *modal voltages* (3). Consider first the electro-optic signal $v(x,z_1)$ as obtained from a single line scan along the x-direction for fixed $z = z_1$. Obviously it is possible to extract the modal voltages $v_i(z_1)$, if the transverse field profiles $u_i(x)$ for all modes which make a non-negligible contribution to the electro-optic signal are known and to a sufficient degree linearly independent over the (finite) measurement interval $x_{\min} \le x \le x_{\max}$.

The transverse distributions $u_i(x)$ can be obtained from a field theoretical analysis and subsequent application of (6). For concise notation let us arrange them into a vector

$$\mathbf{U} := (u_1, u_2, \ldots, u_n)^\mathsf{T} \qquad (7)$$

and furthermore introduce the vector

$$\mathbf{V}(z_1) = (v_1(z_1), v_2(z_1), \ldots, v_n(z_1))^\mathsf{T} \qquad (8)$$

of modal voltages in z_1. In principle then, the determination of the latter reduces to the solution of the matrix equation

$$\langle \mathbf{U}, \mathbf{U}^\mathsf{T} \rangle \mathbf{V}(z_1) = \langle \mathbf{U}, v(x-\xi, z_1) \rangle, \qquad (9)$$

where $\langle , \rangle$ denotes the inner product

$$\langle u, v \rangle := \int_{x=x_{\min}}^{x_{\max}} u(x)v(x)\,\mathrm{d}\,x \qquad (10)$$

(with the obvious dyadic interpretation for vector arguments) and ξ is the offset of the measurement coordinate system with respect to the reference system for $\mathbf{U}$. A slight complication is due to the fact that ξ is only known approximately. Hence it is to be determined simultaneously by minimization of the approximation error

$$\left\| v(x-\xi, z_1) - \mathbf{U}\langle \mathbf{U}, \mathbf{U}^{\mathsf{T}} \rangle^{-1} \langle \mathbf{U}, v(x-\xi, z_1) \rangle \right\|. \quad (11)$$

The value of ξ where (11) attains its minimum is taken for the final solution of (9).

The above procedure amounts to the application of a matched filter for each mode which is considered. Hence reliable extraction of modal voltages is possible even for poor signal to noise ratio and at the limit of the spatial resolution of the measurement system.

Extraction of wave amplitudes

It is possible to extend this approach so as to extract not only the total modal voltages but also decompose them into the complex amplitudes of forward and backward propagating waves. A prerequisite, however, is that the reference plane of interest connects to a section of homogeneous transmission line which is at least $\lambda_i/4$ long. Assume that m linescans have been performed at positions $z_1, \ldots, z_m$ in this interval and subsequently $v_i(z_1), \ldots, v_i(z_m)$ have been extracted as described above. The desired voltage wave amplitudes are then obtained by minimization of

$$\left\| \begin{pmatrix} v_i(z_1) \\ \vdots \\ v_i(z_m) \end{pmatrix} - \begin{pmatrix} e^{-\gamma_i z_1} & e^{\gamma_i z_1} \\ \vdots & \vdots \\ e^{-\gamma_i z_2} & e^{\gamma_i z_2} \end{pmatrix} \begin{pmatrix} v_i^+ \\ v_i^- \end{pmatrix} \right\| \qquad (12)$$

with respect to v_i^+, v_i^-. If the voltages $v_i(z_k)$ were known exactly two linescans would be sufficient. In the presence of measurement errors at least 5 linescans should be available over a length of approximately $\lambda_i/4$. With additional linescans it is possible to treat γ_i as an additional fitting parameter and use field theoretical result as a starting value only.

S-parameter measurement

The complete multimode s-parameter characterization of a multiport component, e.g. a CPW T-junction, including even-odd mode conversion, is merely a formal evaluation of definitions once the amplitudes of forward and backward propagating waves have been extracted for a sufficient set of linearly independent excitations.

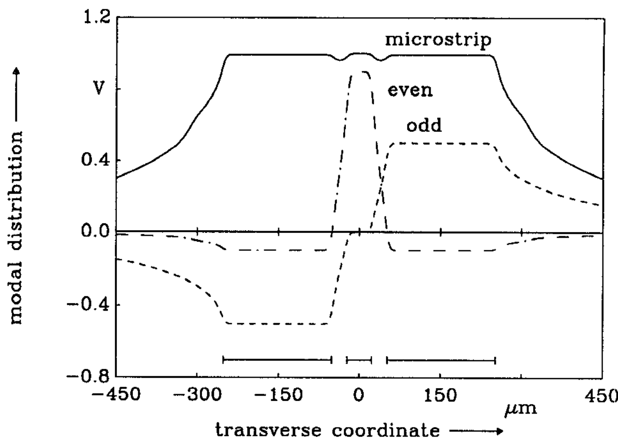

Fig. 3: Computed transverse profiles $u_e(x), u_o(x)$ and $u_{ms}(x)$ for the CPW structure of Fig. 2.

Application to CPW components

We now specialize to the case of CPW interconnects and report on some preliminary experimental results. First thing to discuss is which modes are relevant and how to obtain the correct transverse profiles $u_i(x)$. It was found that accurate specification of the measurement environment is the most important aspect. The three-conductor (ground-center-ground) picture of CPW is an oversimplification because a nearby ground plane is always present in electro-optic probing (but also in a packaged circuit). For predictable results a well defined ground plane must be present and hence the configuration shown in Fig. 2 is chosen as the measurement environment. A sapphire support with a sputtered layer of Indium Tin Oxide is used to provide for a transparent but conducting layer.

Assuming typical dimensions three modes are relevant on the structure shown in Fig. 2: the fundamental CPW even (e) and odd (o) mode and a highly dispersive mode of even symmetry. Though our interest is mainly in even and odd mode amplitudes the latter mode can not be neglected, because it contributes to the electro-optic signal. Its features depend on frequency, substrate dimensions, backplane separation d and total metalization width ($b = w + 2s + 2c$). Limiting cases are the *CPW-microstrip mode* (quasi-static limit), the *CPW surface-wave-like mode* [3, 4] for $d \to \infty$ and for $d, b \to \infty$ it resembles a TM_0 surface wave on a conductor backed dielectric slab. For convenience we shall refer to this mode as the microstrip (ms) mode in all cases, which is justified by its invariably flat transverse profile. With the obvious designations eqn. (5) then takes the form

$$v(x,z) = v_e(z)u_e(x) + v_o(z)u_o(x) + v_{ms}(z)u_{ms}(x) + v_r(x,z). \quad (13)$$

The modal profiles $u_e(x), u_o(x)$ and $u_{ms}(x)$ are obtained from a field theoretical analysis using the boundary integral equation method [5, 6]. For the present investigations

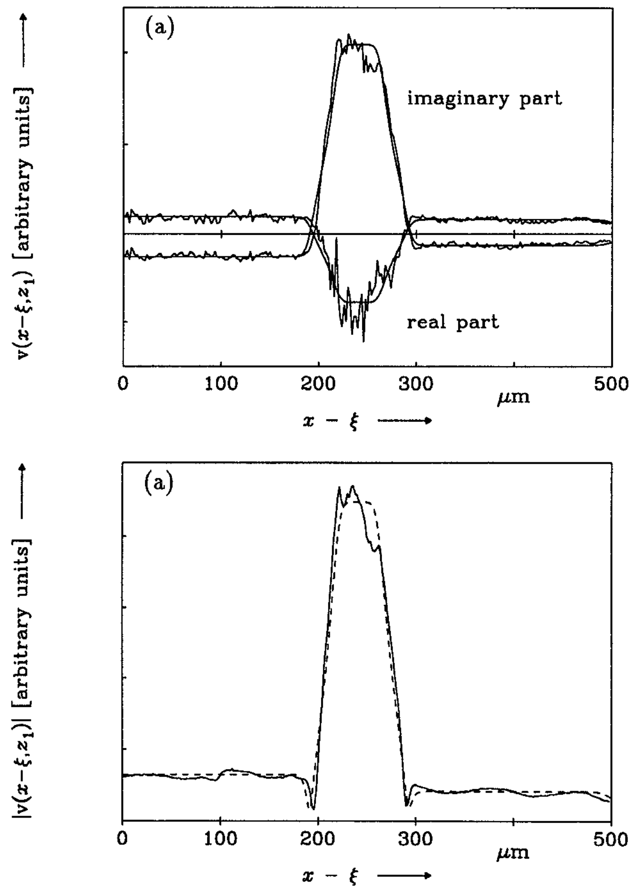

Fig. 4: Comparison of measured transverse profile for the CPW of Fig. 2 at 6 GHz with its approximation in terms of computed profiles (a) real and imaginary parts, (b) magnitude.

we have restricted to quasi-TEM analysis. It is obvious from Fig. 3 that the transverse profiles are linearly independent over the interval $2c + 2s + w$ and can even be made mutually orthogonal by suitable choice of the length of the measurement interval.

Fig. 4 displays the real and imaginary parts of a typical line scan across the same CPW as considered in Fig. 3 together with its approximation as obtained by substitution of the previously extracted modal voltages into (13). The even mode is dominant in this example which was measured at the output port of a coplanar bend, but a non-negligible even-odd mode conversion is visible. It can be seen that the overall match is excellent and only local deviations occur which are mainly due to phase instability of the current measurement setup.

Finally Fig. 5 compares the total even mode voltage at 5 subsequent linescan positions together with their approximation in terms of the extracted amplitudes for forward an backward propagating waves as obtained by minimization of (12). Though these first results are obviously not free from measurement errors it can be concluded that the proposed method promises to be a valuable supplement to conventional on-wafer NWA measurement.

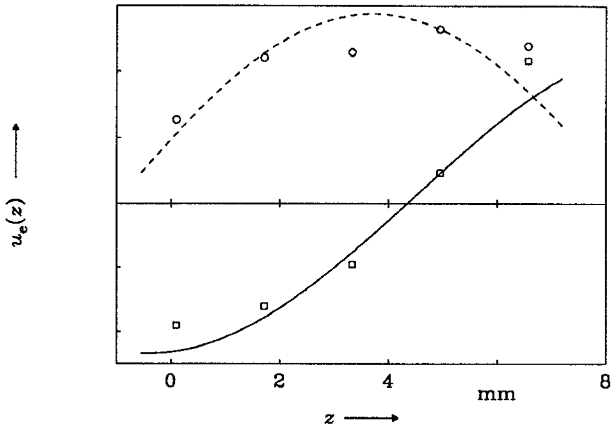

Fig. 5: Real- (squares) and imaginary parts (circles) of extracted even mode voltages at 6 GHz compared to their approximation in terms of forward and backward propagating waves.

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